PAK209 – BJT modelling for Early effect – excerpt

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New BJT modelling for Early effect

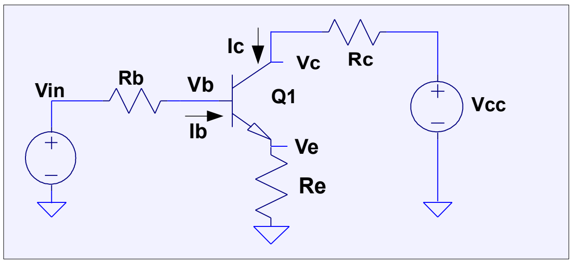

Fig209a.(Fig103a copy) **Basic CE with emitter degeneration**

Thought was given to how to get an exact solution for Ic using the W-approach. The problem is in the SPICE Gummel Poon (GP) model the Early effect uses

$Ic = Ico\,(1+Vce/V_A)$

where Ico is from the Schockley equation and $Vce = Vcc–Ic\cdot R_{TOT}$ which gives

$Ic/(1+(Vcc–Ic\cdot R_{TOT})/V_A) = Ico$.

This gives an Ic term in the denominator which does not allow an exact W-solution.

Instead the original form for the Early effect as used in the improved VBIC [ref BCTM98 text])

$Ic = Ico/(1-Vce/V_E)$ giving

$Ic\,(1-(Vcc–Ic\cdot R_{TOT})/V_E) = Ico$

which requires a solution to $Ic\left(1+b\cdot Ic\right)\, e^{k\,Ic} = e^{\frac{Vin}{Vt}}$. PAK113 allows an approximate solution (see below) with recursion to obtain whatever number of digit accuracy is desired. It turns out the V_E in the above equations needs to be twice the V_{EF} parameter in the LTspice VBIC (level=9) model and V_{EF} maps approximately equal to VAF (V_A above) in the GP model.

Early tests on a TIP31A

A test on a TIP31A (100V 3A NPN) was done to fit to the alternative VBIC Early equation

$Ic = Ico/(1-Vce/V_E)$

Fig.209b is a plot of 1/Ic versus Vce. The Vce intercept V_E is about 110V. For comparison the data was plotted to find the V_A intercept using the standard GP/EM equation and V_A is about 60V or close to half the alternative VBIC Early voltage V_E. Test voltages are Vce of 8V and 32V at 100uA up to 13mA.

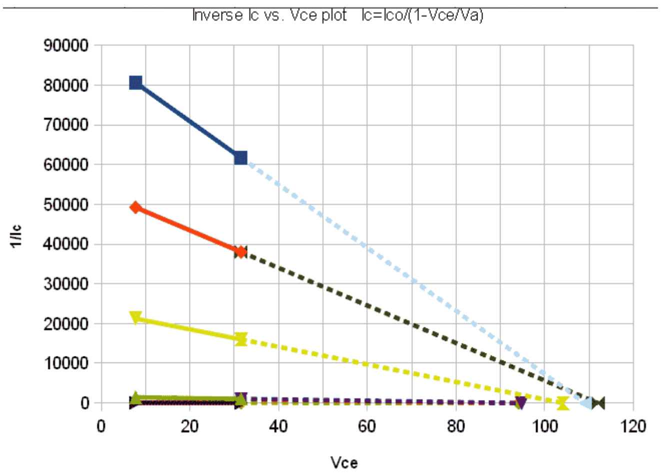

Fig.209b. TIP31A tests plot of 1/Ic versus Vce for the X-intercept V_A

The measured Vceo (base open) breakdown voltage was 130V at 0.25mA. This suggests the alternative VBIC equation could be used to model both the Early effect and Vce breakdown with one parameter (albeit only approximately for Vce_{max} if a best fit is for V_E). Further tests are needed to check typical Vceo breakdown voltages compared to the V_E parameter.

Figure 209c shows plots the currents and derivatives for the GP (standard SPICE) transistor with the Early effect and the VBIC Early effect formulation. The VBIC has some smoothing added seen in the derivative plot around clipping (this may be due to quasisaturation in the VBIC but I could not turn this rounding effect off) which is not modelled in my VBIC Early equation above and is therefore not seen in my Ealy effect solutions below.

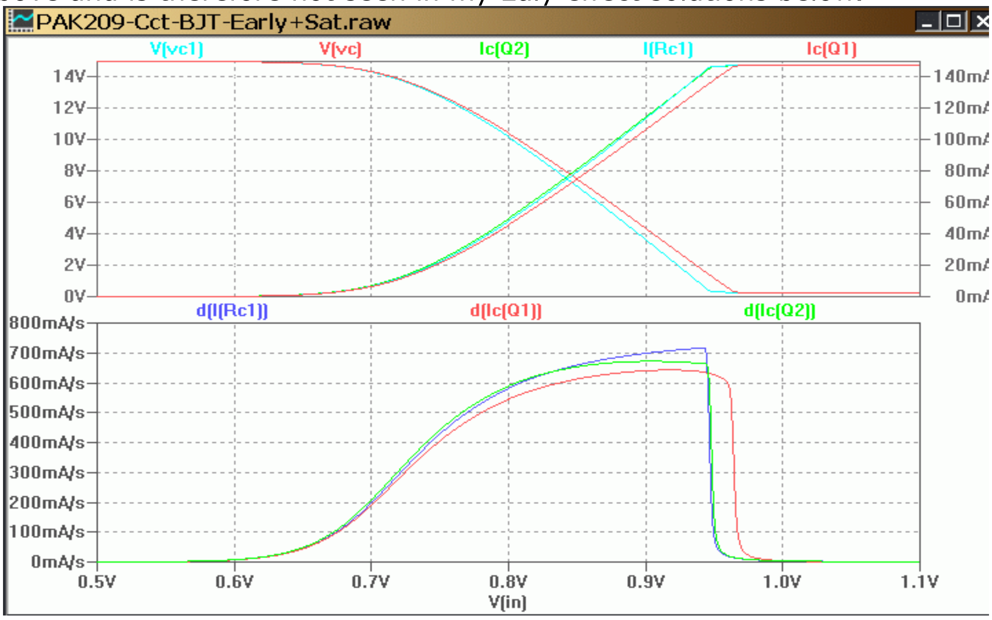

Figure 209c. GP Early effect (Q1)and the VBIC Early effect:
Plot values V_{AF}=V_E=20V, Vcc=15, Is=10e-15, Re=1, Rb=100, Rc=100, Beta= 100.

For these plots I have set VAF to 20V and Vcc to 15V to see the difference clearly. The differences are sensitive to how close the peak Vce gets to V_E in the VBIC formulation since V_E (as mentioned) is also the breakdown voltage (Vce_{max}) and in my model equations using V_E (Veo in the LTspice equations) 'breaks down' at 2*V_{EF} of the VBIC model or 40V the plot above.

New BJT modelling for Early effect excerpt below:

Using the improved VBIC [ref BCTM98 text]) Early effect formulation:

$$Ic = \frac{Iso}{1-\frac{Vce}{V_E}}\left(e^{\frac{Vbe}{Vt}}-1\right)$$ where V_E is the VBIC Early effect (different to EM/GP).

Base $Vbe = Ie\cdot Re + \frac{Ic}{Beta}Rb$ where Beta depends on Vce, $Beta = \frac{Beta_0}{1-\frac{Vce}{V_E}}$ where $Beta_0$ is 'BF'.

$$=> Vbe = Ie\cdot Re + \frac{Ic}{Beta_0}Rb\left(1-\frac{Vce}{V_E}\right) \quad => \quad Vbe = Ic\left(\frac{Beta+1}{Beta}\right)Re + \frac{Ic}{Beta_0}Rb\left(1-\frac{Vce}{V_E}\right) = Ic\cdot Req$$

where $Req = \left(\frac{Beta+1}{Beta}\right)R\mathrm{e} + \frac{Rb}{Beta_0}\left(1-\frac{Vce}{V_E}\right) = R\mathrm{e}' + \frac{Rb}{Beta_0}\left(1-\frac{Vce}{V_E}\right)$ where

$$R\mathrm{e}' = \left(\frac{Beta+1}{Beta}\right)R\mathrm{e} \approx \left(\frac{Beta_0+1}{Beta_0}\right)R\mathrm{e}$$

Substitute Vbe=Vin−Ic·Req gives $Ic = \frac{Is}{1-\frac{Vce}{V_E}}\left(e^{\frac{Vin-Ic\cdot Req}{Vt}}-1\right)$

Substitute $Vce = Vcc - Ic\left(Rc+R\mathrm{e}'\right)$ gives $Ic = \frac{Is}{1-\frac{Vcc-Ic\cdot\left(Rc+R\mathrm{e}'\right)}{V_E}}\left(e^{\frac{Vin-Ic\cdot Req}{Vt}}-1\right)$

Note: Ic(Rc+Re') has Re' because Re' includes the Ib component.

The 'Ic·Req' exponent has a variable Req due to Beta(Vce). The Req term is expanded:

$Req = R\mathrm{e}' + \frac{Rb}{Beta_0}\left(1-\frac{Vcc-Ic\cdot\left(Rc+R\mathrm{e}'\right)}{V_E}\right) = R\mathrm{e}' + \frac{Rb}{Beta_0}\left(1-\frac{Vcc}{V_E}\right) + \frac{Rb}{Beta_0}\frac{Rc+R\mathrm{e}'}{V_E}Ic$ giving

$Req = Req_0\left(1+m\cdot Ic\right)$ where $m = \frac{1}{Req_0}\frac{Rb}{Beta_0}\frac{Rc+R\mathrm{e}'}{V_E}$ and $Req_0 = R\mathrm{e}' + \frac{Rb}{Beta_0}\left(1-\frac{Vcc}{V_E}\right)$

Substitute Req and move the Is terms to LHSPAK200 series p3 of 7

$$Ic\left(1-\frac{Vcc-Ic\cdot\left(Rc+R\mathrm{e}'\right)}{V_E}\right)+Is = Is\cdot e^{\frac{Vin-Ic\cdot Req_0\left(1+m\cdot Ic\right)}{Vt}} \quad =>$$

$$Ic\left(\left(1-\frac{Vcc}{V_E}\right)+\frac{\left(Rc+R\mathrm{e}'\right)}{V_E}Ic\right)+Is = Is\cdot e^{\frac{Vin-Req_0\left(1+m\cdot Ic\right)\cdot Ic}{Vt}}$$

$$Ic\left(1-\frac{Vcc}{V_E}\right)\left(1+\frac{\left(Rc+R\mathrm{e}'\right)Ic}{V_E\left(1-\frac{Vcc}{V_E}\right)}\right)+Is = Is\cdot e^{\frac{Vin-Req_0\left(1+m\cdot Ic\right)\cdot Ic}{Vt}} \quad =>$$

$Ic\left(1-\frac{Vcc}{V_E}\right)\left(1+n\cdot Ic\right)+Is = Is\cdot e^{\frac{Vin-Req_0\left(1+m\cdot Ic\right)\cdot Ic}{Vt}}$ where $n = \frac{Rc+R\mathrm{e}'}{V_E\left(1-\frac{Vcc}{V_E}\right)}$

giving $\left[Ic\left(1+n\cdot Ic\right)+\frac{Is}{1-\frac{Vcc}{V_E}}\right]e^{\frac{Req_0\left(1+m\cdot Ic\right)\cdot Ic}{Vt}} = \frac{Is}{1-\frac{Vcc}{V_E}}\,e^{\frac{Vin}{Vt}}$

It is not possible to solve this exactly for Ic since m and n are not *in general* equal (m=n requires that Re=0). To get a solution we need a (1+m·Ic) term in the LHS mantissa which can be obtained by dividing both sides by (1+n·Ic) then multiplying by (1+m·Ic). This creates a ratio term (1+m·Ic)/(1+n·Ic) on the RHS and this ratio is called a Switch function (Sw). Although an explicit solution cannot be obtained this way, we can use recursion to get whatever accuracy we need. We begin with an estimate Ic_{est} for the RHS ratio of m's and n's that use Ic.

Let $Sw = \frac{1+m\cdot Ic}{1+n\cdot Ic}$. Multiply both sides by (Req_0/Vt)Sw

$$\frac{Req_0}{Vt}\left(Sw\cdot Ic(1+n\cdot Ic)+\frac{Sw\cdot Is}{1-\frac{Vcc}{V_E}}\right)\cdot e^{\frac{Reqo(1+m\cdot Ic)\cdot Ic}{Vt}} = \frac{Sw\, Is\, Req_0}{Vt\left(1-\frac{Vcc}{V_E}\right)}\, e^{\frac{Vin}{Vt}} \quad =>$$

$$\frac{Req_0}{Vt}\left(Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right)\cdot e^{\frac{Reqo\cdot Ic(1+m\cdot Ic)}{Vt}} = \frac{Sw\, Is\, Req_0}{Vt\left(1-\frac{Vcc}{V_E}\right)}\, e^{\frac{Vin}{Vt}}$$

Now add 'Is·Sw' to the LHS and RHS exponents

$$\frac{Req_0}{Vt}\left(Ic(1+m\, Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right)e^{\frac{Req_0}{Vt}\left(Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right)} = \frac{Sw\, Is\, Req_0}{Vt\left(1-\frac{Vcc}{V_E}\right)}\, e^{\left(\frac{Vin+\frac{Sw\cdot Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}}{Vt}\right)}$$

Simplify using Vks (knee voltage defined below) ready for the inverse W-solution

$$\frac{Req_0}{Vt}\left(Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right)\cdot e^{\frac{Reqo\cdot Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}}{Vt}} = e^{\left(\frac{Vin-Vks}{Vt}\right)}$$

where $Vks = Vt\ln\left(\frac{Vt\left(1-\frac{Vcc}{V_E}\right)}{Sw\cdot Is\cdot Req_0}\right)-\frac{Sw\cdot Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$, or $Vks = Vk_0 - Vt\ln(Sw)-\frac{(1-Sw)\cdot Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$

where $Vk_0 = Vt\ln\left(\frac{Vt\left(1-\frac{Vcc}{V_E}\right)}{Is\cdot Req_0}\right)-\frac{Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$ is the constant part of Vks

The W-solution

$$\frac{Req_0}{Vt}\left(Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right) = W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right] \quad =>$$

$$Ic(1+m\cdot Ic)+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)} = \frac{Vt}{Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right] \quad =>$$

$$m\cdot Ic^2+Ic+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}=\frac{Vt}{Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]$$ divide by m (for m≠0, i.e. Rb≠0),

$$Ic^2+\frac{1}{m}Ic+\frac{Sw\cdot Is}{m\left(1-\frac{Vcc}{V_E}\right)}=\frac{Vt}{m\cdot Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]$$ complete the square

$$\left(Ic+\frac{1}{2m}\right)^2=\frac{Vt}{m\cdot Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]+\left(\frac{1}{2m}\right)^2-\frac{Sw\cdot Is}{m\left(1-\frac{Vcc}{V_E}\right)}$$

Square root and solve for Ic for m≠0 (i.e. Rb≠0) is

$$Ic=\frac{-1}{2m}+\sqrt{\left(\frac{1}{2m}\right)^2+\frac{Vt}{m\cdot Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]-\frac{Sw\cdot Is}{m\left(1-\frac{Vcc}{V_E}\right)}}$$

where $m=\frac{1}{Req_0}\frac{Rb}{Beta_0}\frac{Rc+Re}{V_E}$, $n=\frac{Rc+Re'}{V_E\left(1-\frac{Vcc}{V_E}\right)}$, $Sw=\frac{1+m\cdot Ic_{est}}{1+n\cdot Ic_{est}}$ and

$$Vks=Vk_0-Vt\ln(Sw)-\frac{(1-Sw)Is\,Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$$ where $$Vk_0=Vt\ln\left(\frac{Vt\left(1-\frac{Vcc}{V_E}\right)}{Is\cdot Req_0}\right)-\frac{Is\,Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$$ and

$$Req_0=Re'+\frac{Rb}{Beta_0}\left(1-\frac{Vcc}{V_E}\right)\ ,\quad Re'=\left(\frac{Beta+1}{Beta}\right)\cdot Re\approx\left(\frac{Beta_0+1}{Beta_0}\right)\cdot Re$$

The approximation for Ic_{est} in 'Sw' determines the accuracy of the first Ic solution.

A good first approximation for Ic is the solution with Rb=0 (derivation below)

$$Ic_{est}=\frac{Vt}{Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]-\frac{Is}{\left(1-\frac{Vcc}{V_E}\right)}$$ where Vko is given above.

This gives the correct starting knee voltage but Ic_{sat} at clip (Ic_{sat}≈Vcc/(Rc+Re) is too large and this deviation is corrected by further iterations.

How many recursions do I need?

With the Vko starting point for Ic_{est} above the error with the *first* quadratic solution above for Ic was below ~1% when Rb ≈ Beta·Re. So one calculation (other than the Ic_{est} above) is OK for general analytic manipulations. I have chosen to use two recursions in general. But when Re is small (<0.1 ohm for a 100mA small-signal BJT) and if Rb is also small (Rb<Beta·Re) then a third recursion gets less than 1% error (but convergence is very slow in this situation). In LTspice the third recursion can be skipped using a conditional statement
`If(U(Beta*Re-Rb),2nd,1st).`

Error can be checked when Vce=0 which is when $Ic_{sat} = \frac{Vcc}{Rc+Re'}$. With Vce=0 Beta=$Beta_0$

and using the original Schockley equation $Vin_{sat} = Ic_{sat} \cdot Re\left(\frac{Beta_0+1}{Beta_0}\right) + Vt\ln\left(\frac{Ic_{sat}}{Is}+1\right)$.

Then use Vin_{sat} to calculate Ic with Ratio=1. Then calculate the error $\alpha = \frac{Ic - Ic_{sat}}{Ic_{sat}}$ for the first iteration. The second iteration error allows the rate of convergence can be estimated (second error over the first error) for how many recursions are needed to get to your required accuracy. It depends on Re,Rb,Vcc and V_A values, so a `.param` calculation can determine the number of recursions before the .trans Vin sweep. LTspice default Reltol is 0.001 (0.1%). Note the approximation $Re' = Re\left(\frac{Beta_0+1}{Beta_0}\right)$ where $Beta_0$ is used in place of the variable Beta(Vce) may limit the ultimate accuracy unless a correction function is used to account for this. Further work is needed to improve this limit to the ultimate accuracy.

Figure 209d shows the Ic and Vc plots for a Common Emitter amplifier of **Fig.209a** above with Rc=100 ohms Beta=100, V_E=20V, Vcc=15V and Rb and Re are stepped over a wide range. This is a very pleasing solution. All plots are better than 0.1% with two recursions, the exception is Rb=Re=0.1 with ~1% error with 2 recursions.

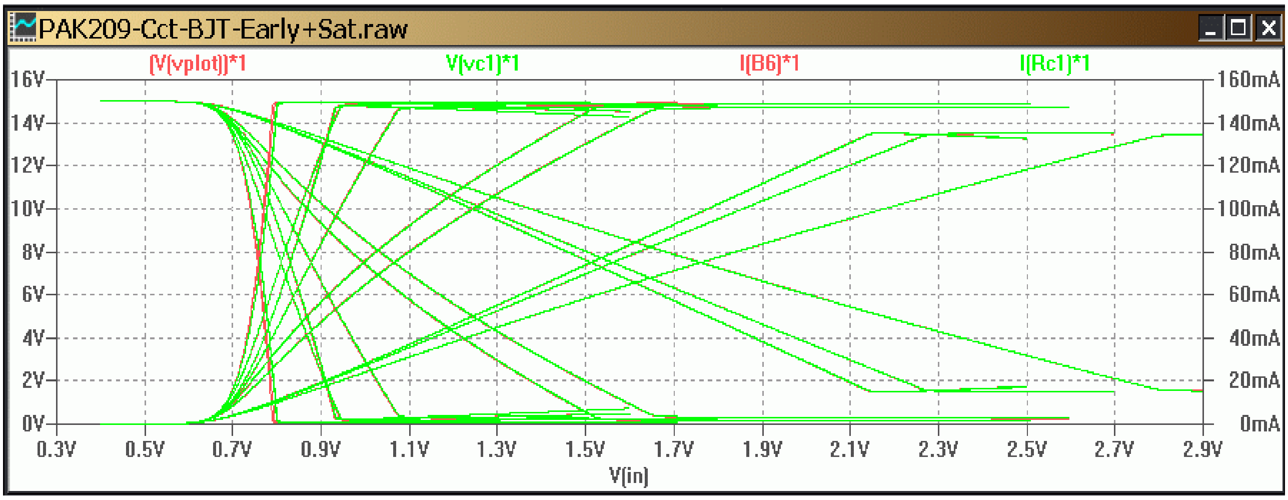

Fig. 209d. W-equations with Early and Saturation using two recursions.
Step Rb 0.1 1 100 500, Re 0.1 1 10, Is=1e-14 Beta=100, V_E=20 Vcc=15 Rc=100.

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Special case: Rb=0

If Rb=0 we get a divide by zero, so for Rb=0 $Req_0 = Re'$, so m=0 giving $Sw = \frac{1}{1+n\cdot Ic}$

Use the alternative W-solution below: Multiply both sides by (Re'/Vt)Sw

$$\frac{Req_0}{Vt}\left(Sw\cdot Ic(1+n\cdot Ic)+\frac{Sw\cdot Is}{1-\frac{Vcc}{V_E}}\right)\cdot e^{\frac{Req_0 Ic}{Vt}} = \frac{Sw\, Is\, Re'}{Vt\left(1-\frac{Vcc}{V_E}\right)} e^{\frac{Vin}{Vt}} \quad =>$$

$$\frac{Req_0}{Vt}\left(Ic+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right)e^{\frac{Req_0 Ic\, Ic}{Vt}} = \frac{Sw\, Is\, Req_0}{Vt\left(1-\frac{Vcc}{V_E}\right)}e^{\left(\frac{Vin+\frac{Sw\cdot Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}}{Vt}\right)}$$ =>

$$\frac{Req_0}{Vt}\left(Ic+\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}\right) = W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]$$ =>

$$Ic = \frac{Vt}{Req_0}W\left[e^{\left(\frac{Vin-Vks}{Vt}\right)}\right]-\frac{Sw\cdot Is}{\left(1-\frac{Vcc}{V_E}\right)}$$ where

$$Sw = \frac{1}{1+n\cdot Ic_{est}}\ ,\ n = \frac{Rc+R\mathrm{e}'}{V_E\left(1-\frac{Vcc}{V_E}\right)}\ ,\ Vks = Vk_0 - Vt\ln(Sw)-\frac{(1-Sw)\, Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}$$ where

$$Vk_0 = Vt\ln\left(\frac{Vt\left(1-\frac{Vcc}{V_E}\right)}{Is\cdot Req_0}\right)-\frac{Is\, Req_0}{\left(1-\frac{Vcc}{V_E}\right)}\ ,\ Req_0 = R\mathrm{e}'+\frac{Rb}{Beta_0}\left(1-\frac{Vcc}{V_E}\right)$$, and

$$R\mathrm{e}' = \left(\frac{Beta+1}{Beta}\right)\cdot R\mathrm{e} \approx \left(\frac{Beta_0+1}{Beta_0}\right)R\mathrm{e}$$

Note: When Rb=0 the same equations can be used $Sw = \frac{1+m\cdot Ic_{est}}{1+n\cdot Ic_{est}}$ (since m=0) and Vks doesn't mind that Rb is 0; the only difference is the main equation for Ic which avoids the divide by zero (m=0). Typical LTspice numerical precision allows Rb to nano-ohms with an Re in the 1 ohm range (small-signal BJT) before needing to consider using the alternative equation for Rb=0.