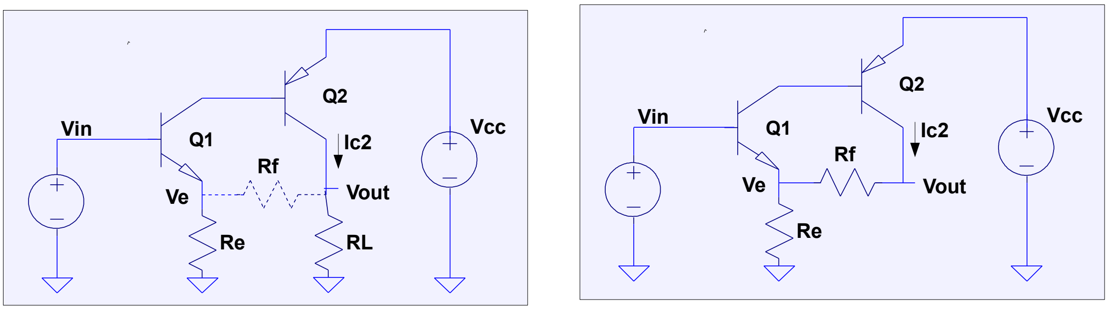
PAK Project – Course Material

PAK30x – Welcome. The W-function analytical approach

 22 Nov 2018 ver 0v1

Level 3 – How to solve multi-transistor circuits

NEW PAK304 – CFP push-pull analysis as a CFP



PAK304a. Complementary Feedback Pair (CFP) Open-Loop [L] and with Rf [R]

Q1 represents the CFA Diamond input stage. Q2 represents the CFA mirror and unity gain buffer with equivalent current gain Beta2≈10,000. Vin drives Q1 with Re degeneration. Q2 drives the output (RL) plus Rf and Re.

Open-Loop

Assume Rf feedback is disconnected and current flows to ground via RL.
PAK 103 solves the Common Emitter amplifier with emitter degeneration Re.

The Shockley equation gives the collector current as

$$I_c = Is\left[e^{\left(\frac{Vbe}{Vt}\right)} - 1\right] \quad => \quad I_c = Is\left[e^{\left(\frac{Vin - I_e \cdot Re}{Vt}\right)} - 1\right]$$

The solution for Q1 is

$$Ic1_{OL} = \frac{Vt}{Req} W\left[e^{\left(\frac{Vin-Vk}{Vt}\right)}\right] - Is_1 \quad \text{where}$$

$$Vk = Vt \cdot \ln\left(\frac{Vt}{Is_1 \cdot Req}\right) - Is_1 \cdot Req \quad \text{and} \quad Req = Re \cdot \left(\frac{1+Beta1}{Beta1}\right)$$

where W[…] is the W-function (a.k.a. Lambert's W-function), Vk is the knee voltage of about 0.6 volts where the transistor current reaches a typical current the mA range. Req is the emitter resistance referred to the collector side since the Schottky equation defines Ic and we want to solve for Ic and not Ie.

Now we know Ic1 in terms of Vin we can calculate Ic2 using $Ic2 = Beta2 \cdot Ic1$.
So Open-Loop solution for Ic2 (output current) is

$$Ic2 = Beta2 \frac{Vt}{Req} W\left[e^{\left(\frac{Vin-Vk}{Vt}\right)}\right] - Beta2 \cdot Is_1$$

The Open-Loop output voltage using Ohm's Law is Vout = Ic2·R_L

Closed-Loop

The Rf feedback is connected and lets assume no current flows to ground via R_L.
The solution for Q1 is now different to the Open-Loop equation because Ic2 now flows through Rf. We can subtract an extra voltage Ic2·Rs from Vin by using Superposition

$Ic1 = \frac{Vt}{Re} W\left[e^{\left(\frac{Vin - Ic2 \cdot \mathrm{Re} - Vk_1}{Vt}\right)}\right] - Is_1$ where Vk, Req are still the same as before.

Applying the W inverse to our new Ic1 equation gives

$$Ic1_{CL} = \frac{Vt}{Req_{CL}} W\left[e^{\frac{Vin - Vk_{CL}}{Vt}}\right] - Is_1 \text{ where}$$

$$Req_{CL} = Re \cdot (Beta2 + 1) \quad \text{and} \quad Vk_{CL} = Vt \cdot \ln\left(\frac{Vt}{Is_1 \cdot Req_{CL}}\right) - Is_1 \cdot Req_{CL}$$

Notice the Req value is multiplied by Beta2, effectively Beta2 times more degeneration for a given resistance. Also the Vk knee voltage is reduced, effectively Q1 turns on 60mV/decade of Beta2 earlier (eg 120mV for a Beta of 100).

Using $Ic1_{CL}$ we calculate Ic2 using $Ic2 = Beta2 \cdot Ic1$.
So the Closed-Loop solution for Ic2 (output current) is

$Ic2_{CL} = Beta2 \frac{Vt}{Req_{CL}} W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right] - Beta2 \cdot Is_1$ where Vk_{CL} and Req_{CL} are given above.

The Beta2 term almost cancels with the Req term giving

$$Ic2_{CL} = \frac{Vt}{Re} W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right] - Beta2 \cdot Is_1$$

The Closed-Loop output voltage using Superposition is

$$Vout = Ic2\left(Rf + \mathrm{Re}\frac{Beta2 + 1}{Beta2}\right)$$

where B2/(B2+1) is from some Ic1 flowing through Re as Ie1.

In terms of Vin Closed-Loop and with no load we get

$$Vout = \left(Rf + \mathrm{Re}\frac{Beta2 + 1}{Beta2}\right)\frac{Vt}{Re} W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right] - Beta2 \cdot Is_1$$

Simplifying and neglecting the Beta2·Is1 term and with B2 large gives

$$Vout \simeq \left(\frac{Rf}{Re} + 1 \cdot \frac{Beta2 + 1}{Beta2}\right) Vt \cdot W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right]$$

With Beta2 large the Ic1 component through Ie is negligible, then

$$Vout \simeq \left(\frac{Rf}{Re} + 1\right) Vt \cdot W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right]$$

The incremental gain dVout/dVin is found by differentiation of the W-function (PAK114) gives

$$\frac{\partial Vout}{\partial Vin} \simeq \left(\frac{Rf}{Re} + 1\right) \frac{W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right]}{1 + W\left[e^{\left(\frac{Vin - Vk_{CL}}{Vt}\right)}\right]}$$

The ratio W[…]/(1+W[…]) reaches 91% of the ultimate Closed-Loop gain of (Rf+Re)/Re when

Vin (the dc bias) is 60mV more than Vk_{CL} .

Notice this is the same ultimate voltage gain for a non-inverting Voltage Feedback amplifier when the feedback loop gain becomes very large.

The ratio 1/(1+W[…] represents the feedback "depth" or the feedback loop gain in dB is 20·Log(1+W[…]). so for say 40dB feedback depth W[…]≈100 which occurs Vin (the dc bias) is 2x60mV higher than Vk_{CL} , or in terms of Ic2≈100·Vt/Re which gives Vre=Ic2·Re≈100·Vt and 100·Vt is 2.6 volts at 300K or 27C. If we chose Re of say 100 ohms then Ic2 would be 2.6/100 amps or 26mA for 40dB of feedback loop gain.

Compare this to the Re method where gm≈Vt/Ic or 26mA/V per mA, giving a gm of 625mA/V at 26mA. The degenerated gm with 100 ohms is 10mA/V so the internal gm is about 62 times higher with about 36dB of loop gain which is close to the 40dB above.

What advantage does the W-solution offer compared to the commonly used small signal $1/r_E$ = gm≈Vt/Ic or 26 Ω·mA method? [ref: Blundell A J, 'New approach to transistor circuit analysis', WW June 1971 p287-292, Pt 2 July 1971 https://www.americanradiohistory.com/Archive-Wireless-World/70s/Wireless-World-1971-06.pdf → p73 of PDF]

In a practical single ended amplifier will be a DC bias plus an AC component. The r_E method requires the DC operating point, then find all the small signal gm's and solve the linearised equations [ref Jarmo Lähdevaara http://www.guitarscience.net/papers/feedback.pdf]

First, using the W-approach we get a large signal solution for IC2 for Closed-Loop feedback and this gives a large signal equation for Vout in terms of the large signal Vin. Once IC2 is known all the other voltages and current s can be found by back substitution. Also the incremental gain at the DC bias point can be known in terms of all the circuit components. In small signal analysis you cannot calculate the DC bias point directly – you have to use approximations, or a graphical method, or use a simulator, or better from a bench test.

Also the 2nd and 3rd harmonic distortion can be calculated in terms of all the circuit components and in terms of the DC bias point – all are variables in the HD2 and HD3 equations so the designer has full information about the circuit currents, voltages, gains and low order distortion components including the thermal effects. PAK117 covers HD2 and HD3 calculation for the CE amplifier with shunt feedback and emitter degeneration.

More sophisticated secondary effects can be added including hitting Vce saturation limits (PAK208), Beta variation with current (PAK208) and the Early effect (PAK209).

High frequency effects cannot be analysed (yet) using W-solutions – HF effects still requires transient simulations using a simulator. Stability analysis using W-analysis using numerical integration offers advantages over the conventional approach [ref: Ulsoy 2009 https://pdfs.semanticscholar.org/5689/dc4c419177a36f2ab1b3726dc6a672b87448.pdf]

In summary, using the W-approach we can have full understanding of how the circuit operates in Open-Loop and Closed-Loop for low frequency signals. With large signal equations we can optimise anything using partial differentiation for maxima/minima's including nulling temperature effects minimising nonlinearity and minimising distortion components.